ch-4 **Expansions**

Some Important Formula

[By: Sonu sir]

(1) $(a+b)^2 = (a)^2 + (b)^2 + 2ab = a^2 + b^2 + 2ab$.

(2.) $(a-b)^2 = (a)^2 + (b)^2 - 2ab = a^2 + b^2 - 2ab$

(3.) $(a+b)(a-b) = a^2 - b^2$ Sonu sir

(4) $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

(5.) $(a+b)^2 - (a-b)^2 = 4ab$

(6.) $\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$ by Sonu Sir

(7) $\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$

(8) $\left(a + \frac{1}{a}\right)^2 + \left(a - \frac{1}{a}\right)^2 = 2\left(a^2 + \frac{1}{a^2}\right)$

(9) $\left(a + \frac{1}{a}\right)^2 - \left(a - \frac{1}{a}\right)^2 = 4$ Sonu Sir

(10) $(a-b) = \sqrt{(a+b)^2 - 4ab}$

(11) $(a+b) = \sqrt{(a-b)^2 + 4ab}$

(12) $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

(13) $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(14.) $(a+b)^3 + (a-b)^3 = 2(a^3)$ [By: Sonu Sir]

(15) $a^3+b^3 = (a+b)^3 - 3ab(a+b)$ → this is Evaluate type

(16) $a^3-b^3 = (a-b)^3 + 3ab(a-b)$

(17) $a^3+b^3 = (a+b)(a^2+b^2-ab)$ → It is Factorise type.

(18) $a^3-b^3 = (a-b)(a^2+b^2+ab)$

(19) $a^3+\frac{1}{a^3} = (a+\frac{1}{a})^3 - 3(a+\frac{1}{a})$

(20) $a^3-\frac{1}{a^3} = (a-\frac{1}{a})^3 + 3(a-\frac{1}{a})$

(21) $a^2+b^2 = (a+b)^2 - 2ab$

(22) $a^2+\frac{1}{a^2} = (a+\frac{1}{a})^2 - 2$

(23) $a^2-\frac{1}{a^2} = (a+\frac{1}{a})(a-\frac{1}{a})$

(24) $a-\frac{1}{a} = \sqrt{(a+\frac{1}{a})^2 - 4}$

(25) $a+\frac{1}{a} = \sqrt{(a-\frac{1}{a})^2 + 4}$

(26) $(a+b)^3 = (a)^3 + (b)^3 + 3a^2b + 3ab^2$

(27) $(a-b)^3 = (a)^3 - (b)^3 - 3a^2.b + 3.a.(b)^2$

(28) $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

by Sonu Sir

φ Evaluate $(998)^2$ (Jonu sir)

[By: Sonu Sir]

Soln $(998)^2 = (1000-2)^2$

Use $\boxed{(a-b)^2 = a^2+b^2-2ab}$

$= (1000)^2 + (2)^2 - 2\cdot 1000 \cdot 2$

$= 1000000 + 4 - 4000$

$= 1000004 - 4000$ Sonu Sir

$= 996004$ Ans

C.W φ Evaluate $(102)^2 = (100+2)^2$

$\boxed{(a+b)^2 = a^2+b^2+2ab}$

$= (100)^2 + (2)^2 + 2\cdot 100 \cdot 2$

$= 10000 + 4 + 400$

$= 10000 + 404$

$= 10404$ Ans

C.W φ Expand $\left(\frac{2x}{7} - \frac{7y}{4}\right)^2$

Soln $\boxed{(a-b)^2 = (a)^2 + (b)^2 - 2ab}$

$= \left(\frac{2x}{7}\right)^2 + \left(\frac{7y}{4}\right)^2 - 2\cdot \frac{2x}{7} \times \frac{7y}{4}$

$= \left(\frac{4x^2}{49} + \frac{49y^2}{16} - xy\right)$ Ans

φ Find Square of $(2a+b)$

Soln $(2a+b)^2 = (2a)^2 + (b)^2 + 2\cdot 2a \cdot b$

$(\because (a+b)^2 = a^2 + b^2 + 2ab)$

$= 4a^2 + b^2 + 4ab$ Ans

Some More Formula →

(29) If $a+b+c=0$

Then $a^3+b^3+c^3=3abc$

Example Q. Use property to Evaluate → $(8)^3+(-5)^3+(-3)^3$

Soln. $(8)^3+(-5)^3+(-3)^3$

Let $a=8$, $b=-5$, $c=-3$

∵ Here $\therefore a+b+c = 8+(-5)+(-3)$

$a+b+c = 8-8$

$\therefore a+b+c=0$

(by sony sir)

$\therefore a^3+b^3+c^3=3abc$

$\therefore (8)^3+(-5)^3+(-3)^3 = 3\cdot 8\times(-5)\times(-3)$

$= 3\times 8\times 15$

$= 3\times 120$

$= 360$

Q. Find square of $(3a+7b)$

Soln. $(3a+7b)^2 = (3a)^2+(7b)^2+2\cdot 3a\cdot 7b$

$= 9a^2+49b^2+42ab$

Soln

* $(a+b)^2 = a^2+b^2+2ab$

$(3a+7b)^2 = (3a)^2+(7b)^2+2\cdot 3a\cdot 7b$

$= 9a^2+49b^2+42ab$

[By: Sonu Sir]

φ Evaluate $(998)^2$

Soln $(998)^2 = (1000-2)^2$

Use $\boxed{(a-b)^2 = a^2+b^2-2ab}$

$= (1000)^2 + (2)^2 - 2\cdot 1000\cdot 2$

$= 1000000 + 4 - 4000$

$= 1000004 - 4000$ Sonu Sir

$= 996004$

C.W φ Evaluate $(102)^2 = (100+2)^2$

$\boxed{(a+b)^2 = a^2+b^2+2ab}$

$= (100)^2 + (2)^2 + 2\cdot 100\cdot 2$

$= 10000 + 4 + 400$

$= 10000 + 404$

$= 10404$

C.W φ Expand $\left(\frac{2x}{7} - \frac{7y}{4}\right)^2$

Soln $\boxed{(a-b)^2 = (a)^2 + (b)^2 - 2ab}$

$= \left(\frac{2x}{7}\right)^2 + \left(\frac{7y}{4}\right)^2 - 2\cdot\frac{2x}{7}\times\frac{7y}{4}$

$= \left(\frac{4x^2}{49} + \frac{49y^2}{16} - xy\right)$

Note
At First Re-solve All Example-type Solved Questions

φ Find Square of $(2a+b)$

Soln $(2a+b)^2 = (2a)^2 + (b)^2 + 2\cdot 2a\cdot b$

$(\because (a+b)^2 = a^2+b^2+2ab)$

$= 4a^2 + b^2 + 4ab$

Ans: The characteristic

* $a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2$

* $\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$

* $a^2 + \frac{1}{a^2} = \left(a - \frac{1}{a}\right)^2 + 2$

* $\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$

Examples

Q If $\left(x - \frac{1}{x}\right) = 3$ Find the value of $\left(x^2 + \frac{1}{x^2}\right)$.

Soln $x - \frac{1}{x} = 3$

S.B.S.

$\left(x - \frac{1}{x}\right)^2 = (3)^2$

$(x)^2 + \left(\frac{1}{x}\right)^2 - 2 \cdot x \cdot \frac{1}{x} = 9$

$x^2 + \frac{1}{x^2} - 2 = 9$

$x^2 + \frac{1}{x^2} = 9 + 2$

$x^2 + \frac{1}{x^2} = 11$

Formula

* $(a-b)^2 = (a+b)^2 - 4ab$

* $(a^2 + b^2) = (a-b)^2 + 2ab$

Q If $a - b = 8$ and $ab = 5$

Find $a^2 + b^2$

Soln $\because a^2 + b^2 = (a-b)^2 + 2ab$

$\therefore a^2 + b^2 = (8)^2 + 2 \times 5$

$= 64 + 10$

$a^2 + b^2 = 74$

Formula

Q Using Formula

$(a+b)^2 = a^2 + b^2 + 2ab$

Evaluate $(999)^2$.

Soln $(999)^2 = (1000 - 1)^2$

$= (1000)^2 + (1)^2 - 2 \cdot 1000 \times 1$

$= 1000000 + 1 - 2000$

$= 1000000 - 1999$

$= 998001$

Formula

Q If $3a + 2b = 23$ and $ab = 20$. Find $(9a^2 + 4b^2)$

Soln $3a + 2b = 23$

S.B.S.

$(3a + 2b)^2 = (23)^2$

$(3a)^2 + (2b)^2 + 2 \cdot 3a \cdot 2b = 529$

$9a^2 + 4b^2 + 12ab = 529$

$9a^2 + 4b^2 + 12 \times 20 = 529$

(10) $a - \frac{1}{a} = 4$

Squaring both side.

$\left(a - \frac{1}{a}\right)^2 = (4)^2$

$(a)^2 + \left(\frac{1}{a}\right)^2 - 2 \cdot a \cdot \frac{1}{a} = 16$

$a^2 + \frac{1}{a^2} - 2 = 16$

$a^2 + \frac{1}{a^2} = 16 + 2$

$\boxed{a^2 + \frac{1}{a^2} = 18}$

$\rightarrow a^2 + \frac{1}{a^2} = 18$

Again S.B.S

$\left(a^2 + \frac{1}{a^2}\right)^2 = (18)^2$

$(a^2)^2 + \left(\frac{1}{a^2}\right)^2 + 2 \cdot a^2 \cdot \frac{1}{a^2} = 324$

$a^4 + \frac{1}{a^4} + 2 = 324$

$a^4 + \frac{1}{a^4} = 324 - 2$

$\boxed{a^4 + \frac{1}{a^4} = 322}$

Sony

* Some more Formulas

(i) $\boxed{(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)}$

(ii) $\boxed{(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca}$

(iii) $\boxed{(a+b-c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca}$

(iv) $\boxed{(a-b-c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc + 2ca}$

Q. Expand.. $(2x + 3y - z)^2$

Q. If $\boxed{a+b+c=4}$ and $\boxed{a^2+b^2+c^2=40}$

Find the value of $(ab + bc + ca)$. Sony

Soln $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$(4)^2 = 40 + 2(ab + bc + ca)$

$16 - 40 = 2(ab + bc + ca)$

$-24 = 2(ab + bc + ca)$

$\rightarrow ab + bc + ca = \frac{-24}{2} = -12$

Q If $a^2+b^2+c^2 = 50$ and $ab+bc+ca = 47$

Find $(a+b+c)$.

Soln $a^2+b^2+c^2 = 50$ and $ab+bc+ca = 47$

Now $\because (a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow (a+b+c)^2 = 50 + 2\times 47$

$\Rightarrow (a+b+c)^2 = 50 + 94$

$\Rightarrow (a+b+c)^2 = 144$

$\Rightarrow a+b+c = \sqrt{144}$

$\therefore a+b+c = 12$

Q If $x+y-z=4$ and $x^2+y^2+z^2 = 30$

Find the value of $(xy-yz-zx)$

Soln $x+y-z=4$ —① and $x^2+y^2+z^2 = 30$

Now $x+y-z=4$

S.B.S.

$(x+y-z)^2 = (4)^2$

$(x+y-z)^2 = x^2+y^2+z^2+2xy-2yz-2zx$

$= x^2+y^2+z^2+2(xy-yz-zx)$

$\Rightarrow x^2+y^2+z^2+2(xy-yz-zx) = 16$

$\Rightarrow 30 + 2(xy-yz-zx) = 16$

$\Rightarrow 2(xy-yz-zx) = 16-30$

$\Rightarrow 2(xy-yz-zx) = -14$

$\Rightarrow xy-yz-zx = -\frac{14}{2} = -7$

(9) CI

Q(1) Calculate the difference bet the C.I. and the S.I. On ₹4000 at 8% p.a. and in 2 years

For C.I. → $P = ₹4000$, $r = 8\%$ p.a., $t = n = 2$ y

$$C.I = P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$$

$$C.I = 4000\left[\left(1+\frac{8}{100}\right)^2 - 1\right]$$

$$C.I = 4000\left[\left(1+\frac{2}{25}\right)^2 - 1\right]$$

$$C.I = 4\times1000\times\left[\left(\frac{25+2}{25}\right)^2 - 1\right]$$

$$C.I. = 4\times1000\times\left[\left(\frac{27}{25}\right)^2 - 1\right]$$

$$C.I. = 4\times1000\times\left(\frac{729}{625} - 1\right)$$

$$C.I = 4\times1000\times\left(\frac{729-625}{625}\right)$$

$$C.I = 4\times\frac{40\,8}{1000}\times\frac{104}{625_{25}}$$

$$C.I = \frac{4\times8\times104}{5} = \frac{3328}{5}$$

$C.I = ₹665.6$ ——— (1)

For S.I

$$S.I = \frac{P\times r\times t}{100}$$

$$S.I = \frac{4000\times8\times2}{100}$$

$S.I = 640$

A/c

Any Six

A/c Diff bet C.I and S.I

$Diff = C.I - S.I = 665.6 - 640$

$Diff = ₹25.6$

Ans