(10) **Remainder and Factor Theorem** [Sonu Kr Gupta]

Class – 10th | Mathematics | class – X th

* Remainder Theorem

(I) If Polynomial $f(x) = ax^3 + bx^2 + cx + d$ divided by Divisor $(x+m)$ then on putting, $x = -m$ in $f(x)$ by Sonu Sir

$$\text{Remainder} = f(-m)$$

$$\text{Remainder} = a(-m)^3 + b(-m)^2 + c(-m) + d$$

(II) If Polynomial $f(x) = ax^3 + bx^2 + cx + d$ divided by Divisor, $(x-m)$ then. Sonu

on putting, $x = m$ in $f(x) \rightarrow R = f(m)$

$$\therefore \text{Remainder} = a(m)^3 + b(m)^2 + c(m) + d$$

(III) If [Polynomial $\rightarrow f(x) = ax^3 + bx^2 + cx + d$] divided by [Divisior $= (nx+m)$] then on putting

$nx + m = 0 \Rightarrow nx = -m \Rightarrow x = -\frac{m}{n}$ in $f(x)$

$$\therefore \text{Remainder} = a\left(-\frac{m}{n}\right)^3 + b\left(-\frac{m}{n}\right)^2 + c\left(-\frac{m}{n}\right) + d$$

* If Polynomial $\rightarrow f(x)$ and Divisior $= (x+m)$

$\therefore$ Remainder $= R = f(-m)$

(II) If Divisior $= (x-m)$

$\therefore R = f(m)$

(III) If Divisior $= (nx+m)$

$\therefore R = f\left(-\frac{m}{n}\right)$

Sonu

Example → (1) Find the Remainder on dividing $f(x) = 3x^3 - 7x^2 + 11x + 1$, by $(x+3)$.

Soln ∵ Polynomial = $f(x) = 3x^3 - 7x^2 + 11x + 1$

∵ Divisior = $x + 3$

∴ by Remainder theorem, Putting, $x = -m$

or $x = -3$ in $f(x)$

$R = f(-3)$

∴ Remainder = $3(-3)^3 - 7(-3)^2 + 11(-3) + 1$

$= 3(-27) - 7 \times 9 - 33 + 1$

$= -81 - 63 - 33 + 1$

$= -177 + 1$

Remainder = -176 Ans

Q. If two Polynomial $P(x)$ and $q(x)$ divided by same Divisior $(x+m)$ and remainder be same then on putting, $x = -m$ in both $P(x)$ and $q(x)$ such that

$$P(-m) = q(-m)$$

After putting the value as above and simplify we will get Un-known Value in given Questions.

(10)

(Q) Using Remainder theorem if $f(x) = 3x^3 + 7x^2 - 5x + 1$ Divided by $(x+3)$.

(Sonu Kr Gupta)

Soln.

$$f(x) = 3x^3 + 7x^2 - 5x + 1$$

$\because$ Divisior $= x + 3$

$\therefore$ putting, $\boxed{x = -3}$ in $f(x)$

$\therefore$ Remainder $= f(-3)$

$$= 3(-3)^3 + 7(-3)^2 - 5(-3) + 1$$

$$= 3 \times (-27) + 7 \times 9 + 15 + 1$$

$$= -81 + 63 + 16$$

$$= -81 + 79$$

Sonu

$$\boxed{\text{Remainder} = -2}$$

Q Find "a" if the polynomials $(ax^3 + 3x^2 - 9)$ and $(2x^3 + 4x + a)$, Leaves the same remainder When divided by $(x+3)$.

Sonu

Soln Let $\boxed{p(x) = ax^3 + 3x^2 - 9}$,

and $\boxed{q(x) = 2x^3 + 4x + a}$

$\because$ Divisior $= x + 3$

$\therefore$ putting $\boxed{x = -3}$

$\therefore$ A/Q $\boxed{p(-3) = q(-3)}$

$$a(-3)^3 + 3(-3)^2 - 9 = 2(-3)^3 + 4(-3) + a$$

$$a(-27) + 3 \times 9 - 9 = 2(-27) - 12 + a$$

$$-27a + 27 - 9 = -54 - 12 + a$$

$$-27a + 18 = -66 + a$$

$$-27a - a = -66 - 18$$

$$-28a = -84$$

$$a = \frac{84}{28} = 3$$

$$(a = 3)$$

(10) **Factor Theorem** → If a polynomial $f(x)$ has Factor $(x+m)$ then on putting $x = -m$ remainder must be zero → $R = 0$

$$\therefore f(-m) = 0$$

for example → If $f(x) = ax^3 + bx^2 + cx + d$

and Factor $= x + m$

$\therefore$ On Putting, $x = -m$

$\therefore f(-m) = 0$

$$\therefore a(-m)^3 + b(-m)^2 + c(-m) + d = 0$$

by Sonu Sir

(II) If Factor $= (x - m)$

then on putting, $x = m$ in $f(x)$ and remainder $= 0$

and $f(m) = 0$.

$$\therefore a(m)^3 + b(m)^2 + c(m) + d = 0$$

by Sonu Kr. Gupta

Q Show that $(x-3)$ is a factor of polynomial

$$x^3 - 7x^2 + 15x - 9$$

Sol Let $f(x) = x^3 - 7x^2 + 15x - 9$

Factor $= x - 3$

$\therefore$ putting, $x = 3$

$\therefore f(3) = (3)^3 - 7(3)^2 + 15(3) - 9$

$= 27 - 7 \times 9 + 45 - 9$

$= 72 - 63 - 9$

$= 72 - 72$

$$f(3) = 0$$

$\therefore$ yes, $(x-3)$ is a factor

(8) Let

$f(x) = ax^3 + 9x^2 + 4x - 10$

Divisior $= (x+3)$

remainder $= R = 5$

$\therefore$ on putting, $x = -3 \rightarrow f(x)$

$\therefore f(-3) = R$

$\therefore a(-3)^3 + 9(-3)^2 + 4(-3) - 10 = 5$

$a(-27) + 9 \times 9 - 12 - 10 = 5$

$-27a + 81 - 22 = 5$

$-27a = 5 + 22 - 81$

$-27a = 27 - 81$

$+27a = +54$

$a = 2$

Ans

If Factor is given, Find Un-known element in Polynomial

Q. If $(3x-2)$ is a Factor of $(3x^3 - kx^2 + 21x - 10)$. Find k.

Soln $f(x) = 3x^3 - kx^2 + 21x - 10$

$\therefore$ Factor $= (3x-2) \rightarrow \therefore$ putting, $3x - 2 = 0$

$3x = 2$

$x = \frac{2}{3}$

$\therefore$ by Factor theorm

$$f\left(\frac{2}{3}\right) = 0$$

$$\therefore 3\left(\frac{2}{3}\right)^3 - k\left(\frac{2}{3}\right)^2 + 21 \times \frac{2}{3} - 10 = 0$$

$$3 \times \frac{8}{27_9} - k \cdot \frac{4}{9} + 14 - 10 = 0$$

$$\frac{8}{9} - \frac{4k}{9} + 4 = 0$$

$$\Rightarrow \frac{8 - 4k + 36}{9} = 0$$

$$-4k + 44 = 0 \times 9$$

$$-4k = +44$$

$$k = \frac{44}{4}$$

$k = 11$

Ans

(19) (19) Use Factor theorem Factorise $2x^3+x^2-13x+6$

Soln Let $f(x) = 2x^3+x^2-13x+6$

Put $x=2$ $\rightarrow$ $f(2) = 2(2)^3+(2)^2-13\times2+6$

$f(2) = 2\times8+4-26+6$

$f(2) = 16+4-26+6$

$f(2) = 26-26$

$f(2)=0$

$\therefore (x-2)$ is a Factor of $f(x)$

Now, Divide the given Polynomial to Factorise

$$x-2 \;\Big)\; 2x^3+x^2-13x+6 \;\Big(\; 2x^2+5x-3$$

$2x^3-4x^2$ (+)

$\times \quad 5x^2-13x+6$

$5x^2-10x$ (−)

$\times \quad -3x+6$

$-3x+6$ (+)

$\times \quad \times$

Done

$\therefore 2x^3+x^2-13x+6 = (x-2)(2x^2+5x-3)$

$= (x-2)[2x^2+6x-x-3]$

$= (x-2)[2x(x+3)-1(x+3)]$

$= (x-2)(x+3)(2x-1)$

To Find Un-Known element and Also Factorise the Polynomial with puts Un-Known element ↓ Sony

Q if $(x-2)$ is a Factor of $2x^3 - x^2 - px - 2$ then Find Value of 'p' and with Value of 'p' factorise the above

Sol: Let $f(x) = 2x^3 - x^2 - px - 2$

$\therefore$ Factor $= (x-2)$

$\therefore$ putting $x = +2$ in $f(x)$

by Factor theorem $f(2) = 0$

$\therefore 2(2)^3 - (2)^2 - p(2) - 2 = 0$

$2 \times 8 - 4 - 2p - 2 = 0$

$16 - 4 - 2p - 2 = 0$

$-2p + 10 = 0$

$-2p = -10$

$p = \frac{-10}{-2}$

$p = +5$

Now put the Value of $p = +5$ in $f(x)$.

Now $f(x) = 2x^3 - x^2 - 5x - 2$ for Factorise, Now Divide by $(x-2)$

$$
\begin{array}{r|l}
x-2 & 2x^3 - x^2 - 5x - 2 \\
 & \underline{2x^3 \mp 4x^2} \\
 & \times \quad 3x^2 - 5x - 2 \\
 & \underline{3x^2 \mp 6x} \\
 & \times \quad x - 2 \\
 & \underline{x \mp 2} \\
 & \times \quad \times
\end{array}
\qquad 2x^2 + 3x + 1
$$

$\therefore 2x^3 - x^2 - 5x - 2 = (x-2)(2x^2 + 3x + 1)$

$= (x-2)(2x^2 + 2x + x + 1)$

$= (x-2)[2x(x+1) + 1(x+1)]$

$= (x-2)(x+1)(2x+1)$

Sony

To Find Un-Known element and Also Factorise the Polynomial with puts Un-Known element ↓ Jony

Q if $(x-2)$ is a Factor of $2x^3 - x^2 - px - 2$ then. Find Value of 'p' and with Value of 'p' factorise the above

Sol: Let $f(x) = 2x^3 - x^2 - px - 2$

$\because$ Factor $= (x-2)$

$\therefore$ putting $x = +2$ in $f(x)$

by Factor theorem $f(2) = 0$

$\therefore 2(2)^3 - (2)^2 - p(2) - 2 = 0$

$2 \times 8 - 4 - 2p - 2 = 0$

$16 - 4 - 2p - 2 = 0$

$-2p + 10 = 0$

$-2p = -10$

$p = \frac{-10}{-2}$

$p = +5$

Now put the Value of $p = +5$ in $f(x)$.

Now $f(x) = 2x^3 - x^2 - 5x - 2$ for Factorise, Now Divide by $(x-2)$

$$
\begin{array}{l}
x-2 \;\big)\; 2x^3 - x^2 - 5x - 2 \;\big(\; 2x^2 + 3x + 1 \\
\quad \underline{-2x^3 \mp 4x^2} \\
\quad\quad \times \quad 3x^2 - 5x - 2 \\
\quad\quad \underline{-3x^2 \mp 6x} \\
\quad\quad\quad \times \quad x - 2 \\
\quad\quad\quad \underline{-x \mp 2} \\
\quad\quad\quad\quad \times \;\; \times
\end{array}
$$

$\therefore 2x^3 - x^2 - 5x - 2 = (x-2)(2x^2 + 3x + 1)$

$= (x-2)(2x^2 + 2x + x + 1)$

$= (x-2)[2x(x+1) + 1(x+1)]$

$= (x-2)(x+1)(2x+1)$

Jony

(19) Use Factor theorem Factorise $2x^3+x^2-13x+6$

(19) Soln Let $f(x) = 2x^3+x^2-13x+6$

Put $x=2$ $\rightarrow$ $f(2) = 2(2)^3+(2)^2-13\times2+6$

$f(2) = 2\times8+4-26+6$

$f(2) = 16+4-26+6$

$f(2) = 26-26$

$f(2)=0$

$\therefore$ $(x-2)$ is a Factor of $f(x)$

Now, Divide the given Polynomial to Factorise

$$x-2\ \big)\ 2x^3+x^2-13x+6\ \big(\ 2x^2+5x-3$$

$$2x^3-4x^2$$

$$5x^2-13x+6$$

$$5x^2-10x$$

$$-3x+6$$

$$-3x+6$$

$$\times\quad\times$$

$\therefore$ $2x^3+x^2-13x+6 = (x-2)(2x^2+5x-3)$

$= (x-2)[2x^2+6x-x-3]$

$= (x-2)[2x(x+3)-1(x+3)]$

$= (x-2)(x+3)(2x-1)$

Q Using R.T. Find the value of 'k' if on dividing $(2x^3 + 3x^2 - kx + 5)$ by $(x-2)$ leaves a remainder 7.

Sol^n Let $f(x) = 2x^3 + 3x^2 - kx + 5$

Divisior = $(x-2)$

and Remainder = $\boxed{R = 7}$

$\therefore$ On putting, $\boxed{x = 2}$ in $f(x)$

A/c $f(2) = R$

$\therefore$ $2(2)^3 + 3(2)^2 - k \times 2 + 5 = 7$

$\Rightarrow 2 \times 8 + 3 \times 4 - 2k + 5 = 7$

$\Rightarrow 16 + 12 - 2k + 5 = 7$

$\Rightarrow 28 - 2k = 7 - 5$

$\Rightarrow 28 - 2k = 2$

$\Rightarrow -2k = 2 - 28$

$\Rightarrow -2k = -26$

$\Rightarrow k = \frac{-26}{-2}$ 13

$\boxed{k = 13}$

Note. Write down all theories explain by Notes and try to solve Following Questions ↓

Q.N. (1) Find the remainder When polynomial $f(x) = 3x^3 - 7x^2 + 11x + 1$ divided by $(x+3)$

Ans. -176

Q.N. (2) Using Remainder theorem, Find the remainder When $f(x) = 3x^3 + 7x^2 - 5x + 1$ Divided by $(x-2)$.

Q.N. (3) Using Remainder theorem, Find the value of K If on dividing $(2x^3 + 3x^2 - Kx + 5)$ by $(x-2)$, Leaves remainder "7".

Ans. $K = 13$

Q.N(4) If $(x+1)$ is a Factor of $(3x^3 + Kx^2 + 7x + 4)$ then Find the value of K

[Ans. $\Rightarrow K = 6$]

Q.N (5) If $(x-2)$ is a Factor of $(2x^3 - x^2 - px - 2)$ then Find the value of p.

[Ans $P = 5$]

Q.N (6) If $(2x+1)$ is a Factor of $(6x^3 + 5x^2 + ax - 2)$ Find the value of "a"

[Ans. $a = -3$]

Q.N.(7) Using the Remainder and Factor Theorem Factorise (i) $x^3 + 10x^2 - 37x + 26$

(ii) $2x^3 + x^2 - 13x + 6$

(iii) $x^3 + 2x^2 - 5x - 6$

by Sonu Sir

Note At First Read and Write all theories and Examples